INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

SHAPE OPTIMIZATION AND IDENTIFICATION OF TEXTILE STRUCTURES DURING TRANSIENT HEAT TRANSFER

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PROJEKT OPTIS PRO FT, reg. č.: CZ.1.07/2.2.00/28.0312 JE SPOLUFINANCOVÁN EVROPSKÝM SOCIÁLNÍM FONDEM
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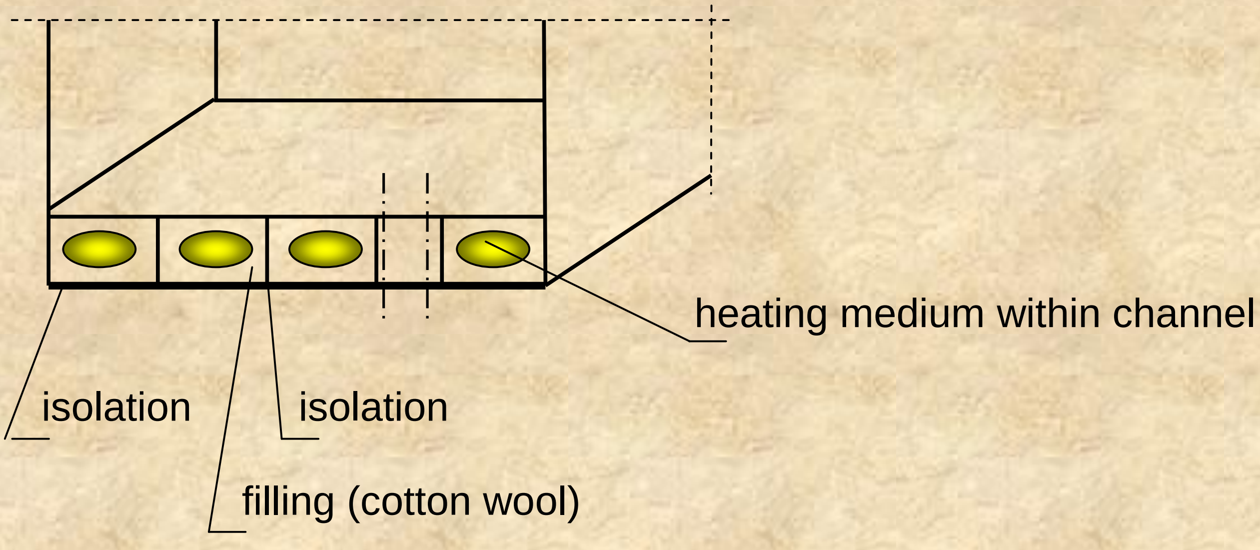
Heat transport in textile structures and products

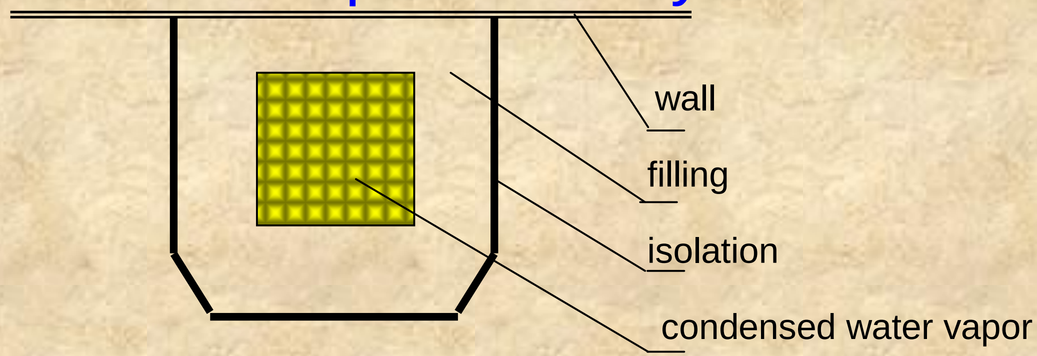
- Outerwear / Protective clothing made of the multilayer composite fabric,
- Fireman suit during the firefighting action,
- Tent protecting against the weather conditions,
- Heating floor panels with the textile filling,
- Feed pipes securing the water vapor transport to the dryer / outlet pipes transporting the condensed water vapor from the dryer,
- Shrinking device to the knitted fabric,
- Outlet pipes transporting the water vapor from the shrinking device,
- Etc.

Motivations – practical problems

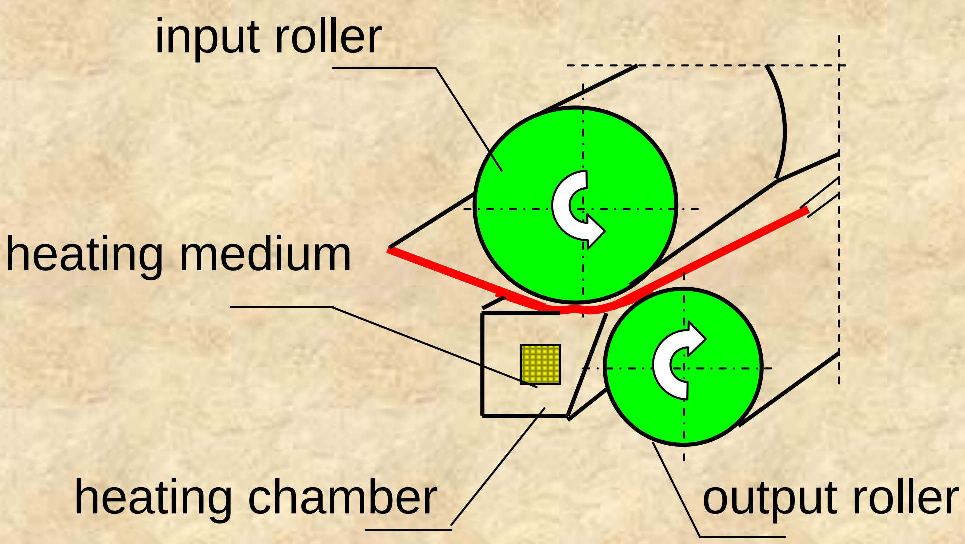
Heating floor panels with textile filling



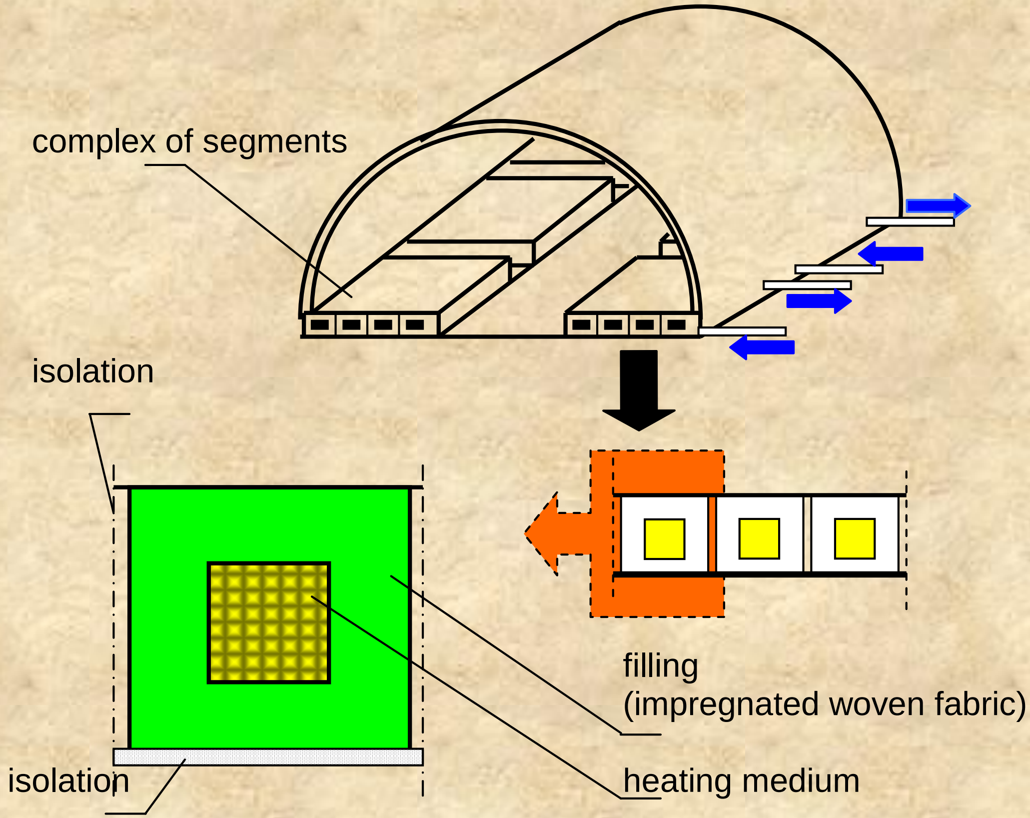
Outlet pipe transporting the condensed water vapor from dryer



Heating chamber within shrinking device to knitted fabric



Tent protecting against weather conditions



Assumptions

- The transient heat transfer problem is defined by the heat transport equation and the set of boundary and initial conditions.
- Both physical and mathematical models of the real structure should be introduced.
- The material derivative concept i.e. the first-order sensitivity of the objective functional with respect to the design parameter can be applied.

Solution strategy of shape optimization / shape identification problem

Primary problem. Initial conditions of heat transport.
Vector of design variables.

↓

Physical model. Definitions of material parameters. Homogenization.

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Mathematical model. General form of objective functional.
Determination of state variables for primary problem.

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Sensitivity analysis. Determination of state variables and sensitivity expressions by means of direct and adjoint approaches.

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Shape optimization / identification. Particular form of objective functional.
Numerical solution.

↓

Physical interpretation. Visualization = distribution maps of state variables.

State variables vs. design variables

- State variables describe the state of the dynamical system / the textile structure during the heat transfer.
  Thermal problems = temperature T
- Design variables determine the structure from the optimization point of view. Design variabels are for example:
  - the coordinates of the crucial structural points,
  - the material thickness,
  - the material parameters, etc.
  Design variables can be continuous (cf. the thickness), discrete (cf. the coordinates of the points) or boolean (cf. the number of layers within the complex tetile structure).

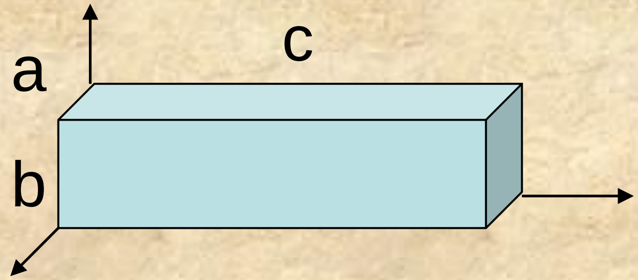
Physical model. Homogenization

Textiles (woven fabrics, knitted fabrics, nonwovens) have:
- inhomogeneous, repeatable structure,
- composite structure: matrix of fibers within the filling.

Homogenization of textile structures
- creation of the homogeneous, orthotropic structure,
- average thermal and mass transfer conductivity coefficients for different materials of matrix and filling,

- rule of mixture (Golanski,Terada,Kikuchi 1997)
- hydrostatic analogy by means of the Turner's Model (G, T, K, 1997)
- proportional volume fraction for composite materials (Tomeczek 1999)
- other metods (cf. solutions for porous structures, composites, etc.)

Physical model. Homogenization



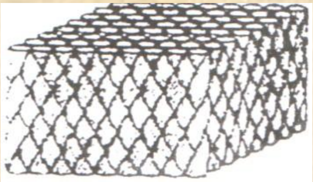
Problem of homogenization scale (micro- and macroscale)

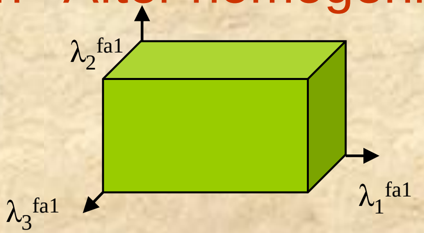
Comparable dimensions a, b, c - 3D homogeneous textile structure
Negligible one dimension - 2D homogeneous textile structure

Basic textile structures (yarn, ropes) - macroscale: 1D or microscale: 3D
Flat textile products - macroscale: 2D or microscale: 3D

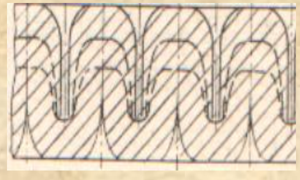
Physical model. Homogenization

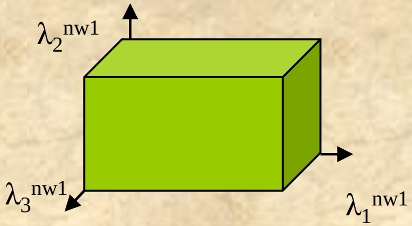
Before homogenization After homogenization



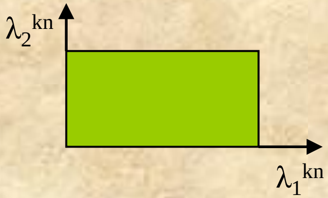


3D plaiting fabric

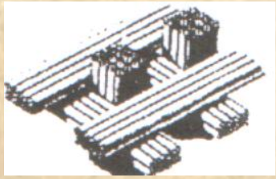


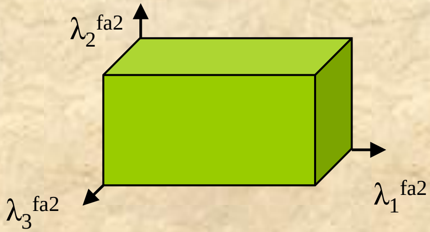


3D nonwovens



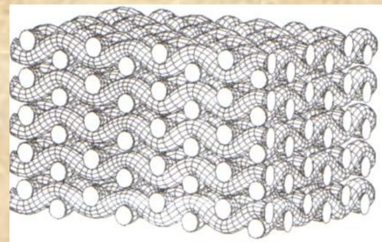
2D knitted fabric





3D system of fabric

Mathematical model

T_1 →  → $T_2<T_1$

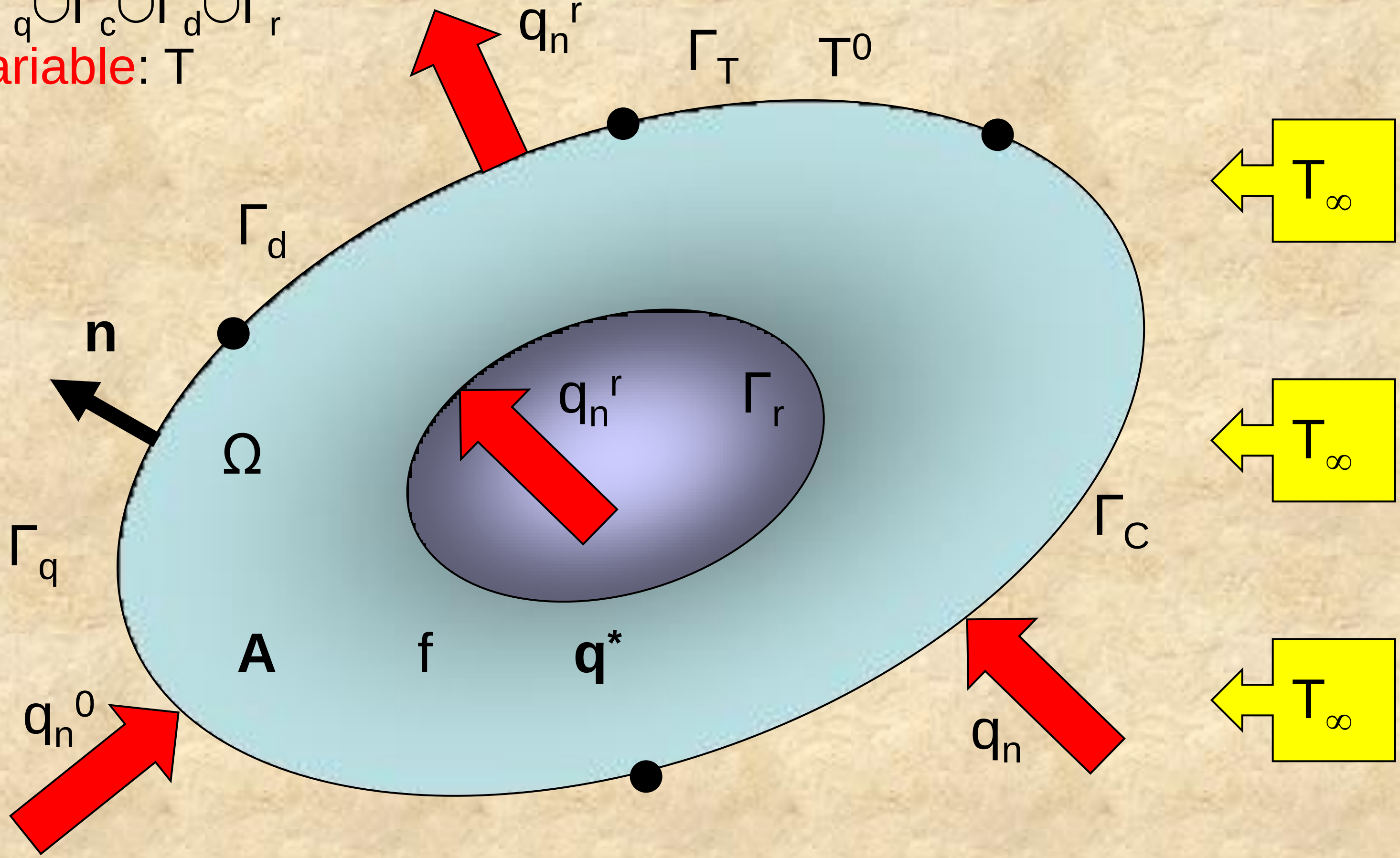
- Fabric packed with fibers, subjected to heat transport (i.e. the temperature gradient).
- State variable: T.
- Fourier's Law: the transient heat transfer within the material is proportional to the temperature gradient and to the area.

Mathematical model. Primary heat conduction model

anisotropic body Ω

$\Gamma = \Gamma_T \cup \Gamma_q \cup \Gamma_c \cup \Gamma_d \cup \Gamma_r$

State variable: T



Mathematical model. Primary heat conduction model

Differential transport equation:

$$\left.\begin{array}{l} -\operatorname{div}\mathbf{q} + f = c\dot{T} \\ \mathbf{q} = \mathbf{A}\cdot\nabla T + \mathbf{q}^{*} \end{array}\right\} \text{within } \Omega; \quad \dot{T} = \frac{dT}{dt}$$

Dirichlet condition: $T(\mathbf{x},t) = T^{o} \text{ on } \Gamma_T$

Neumann condition: $q_n(\mathbf{x},t) = \mathbf{n}\cdot\mathbf{q} = q_n^{o} \text{ on } \Gamma_q$

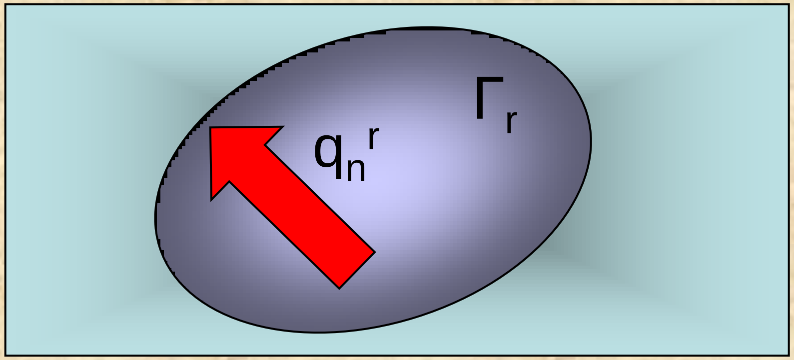
Newton condition: $q_n(\mathbf{x},t) = h\left[T - T_\infty\right] \text{on } \Gamma_c$

Radiation condition: $\mathbf{n}\cdot\mathbf{q}^{r}(\mathbf{x},t) = q_n^{r}(\mathbf{x},t) \text{ on } \Gamma_r$

Radiation condition: $q_n^{r}(\mathbf{x},t) = \sigma T(\mathbf{x},t)^4 \text{ on } \Gamma_d$

Initial condition: $T(\mathbf{x},0) = T_0 \text{ on } \Omega \cup \Gamma$

Mathematical model. Radiation condition according Bialecki



- Closed and convex shape of the hole,
- Non-gray medium within the hole,
  i.e. participating in radiation process,
- For simplicity absorption coefficient = constant, i.e. medium is isothermal and participating in radiation process,
- Radiation properties can be additionally:
  - wavelength independent: the gray wall
  - direction of incoming radiation independent: the diffuse wall

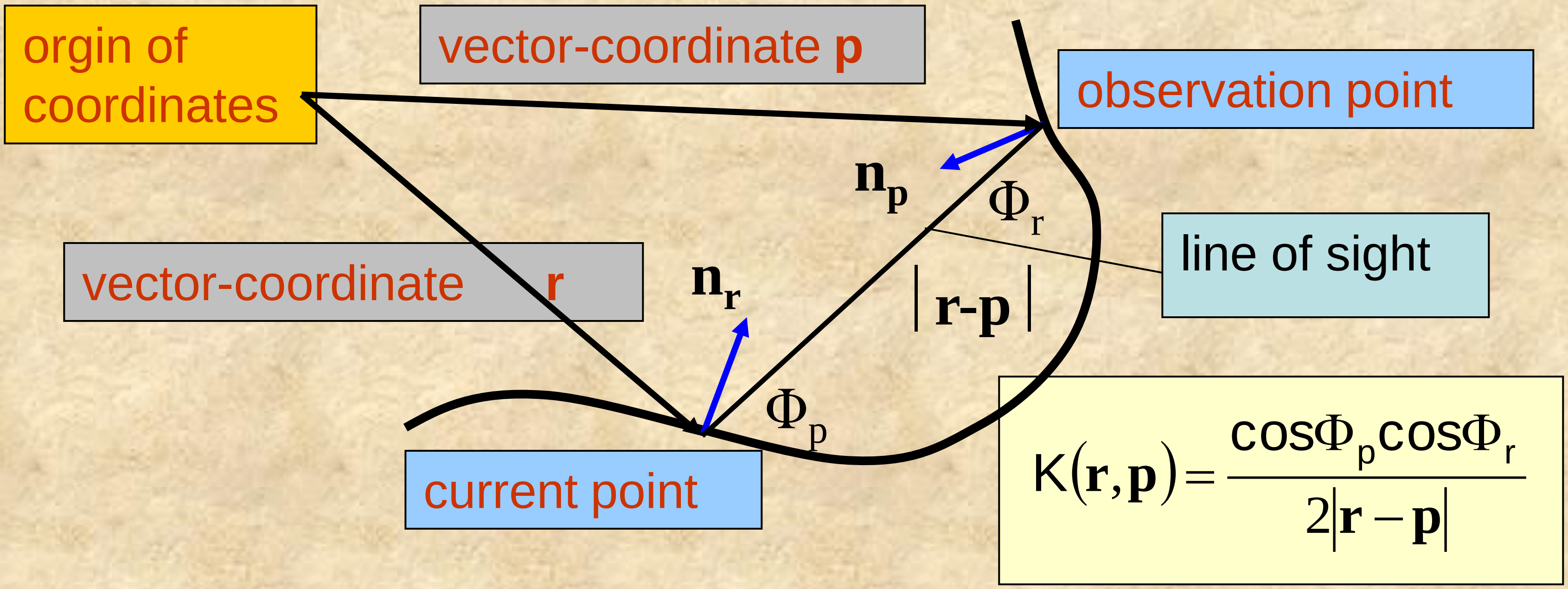
Mathematical model. Radiation condition according Bialecki

$$q_n^r(\mathbf{p}) + \varepsilon(\mathbf{p})\, e_b[T(\mathbf{p})] = \varepsilon(\mathbf{p}) \int_{\Gamma_r} \left\{ e_b[T(\mathbf{r})] + \frac{1-\varepsilon(\mathbf{r})}{\varepsilon(\mathbf{r})} q_n^r(\mathbf{r}) \right\} K(\mathbf{r},\mathbf{p}) \exp(-a|\mathbf{r}-\mathbf{p}|)\, d\Gamma_r + \varepsilon(\mathbf{p})\, e_b(T^m) \int_{\Gamma_r} K(\mathbf{r},\mathbf{p}) [1 - \exp(-a|\mathbf{r}-\mathbf{p}|)]\, d\Gamma_r$$

p, **r**	vector coordinates of the observation point and the current point, respectively,
ε	total hemispherical emissivity of the surface
$e_b[T(\mathbf{p})]$	blackbody emissive power (Stefan-Boltzmann Law)
a	absorption coefficient,
$K(\mathbf{r},\mathbf{p})$	kernel function,
|r-p|	distance between the current and observation points.

Left-hand side = observation point: vector **p**
Right-hand side = current point: vector **r**
1st integral: radiation of walls; 2nd integral: radiation of medium

Mathematical model. Radiation condition according Bialecki



Φ_p Φ_r angles between line of sight and normal to the surface directed outwards on Γ, in the observation and current points, respectively

Mathematical model. Optional objective functional

The optional objective functional

$$F = \int_0^{t_f} \left[\int_\Omega \Psi(T, \nabla T, \mathbf{q}, f, \dot{T}) \, d\Omega + \int_\Gamma \gamma(T, q_n, T_\infty) \, d\Gamma \right] dt$$

Ψ, γ continuous and differentiable functions of the arguments

Material derivative concept:

1st order sensitivity with respect to design parameter:

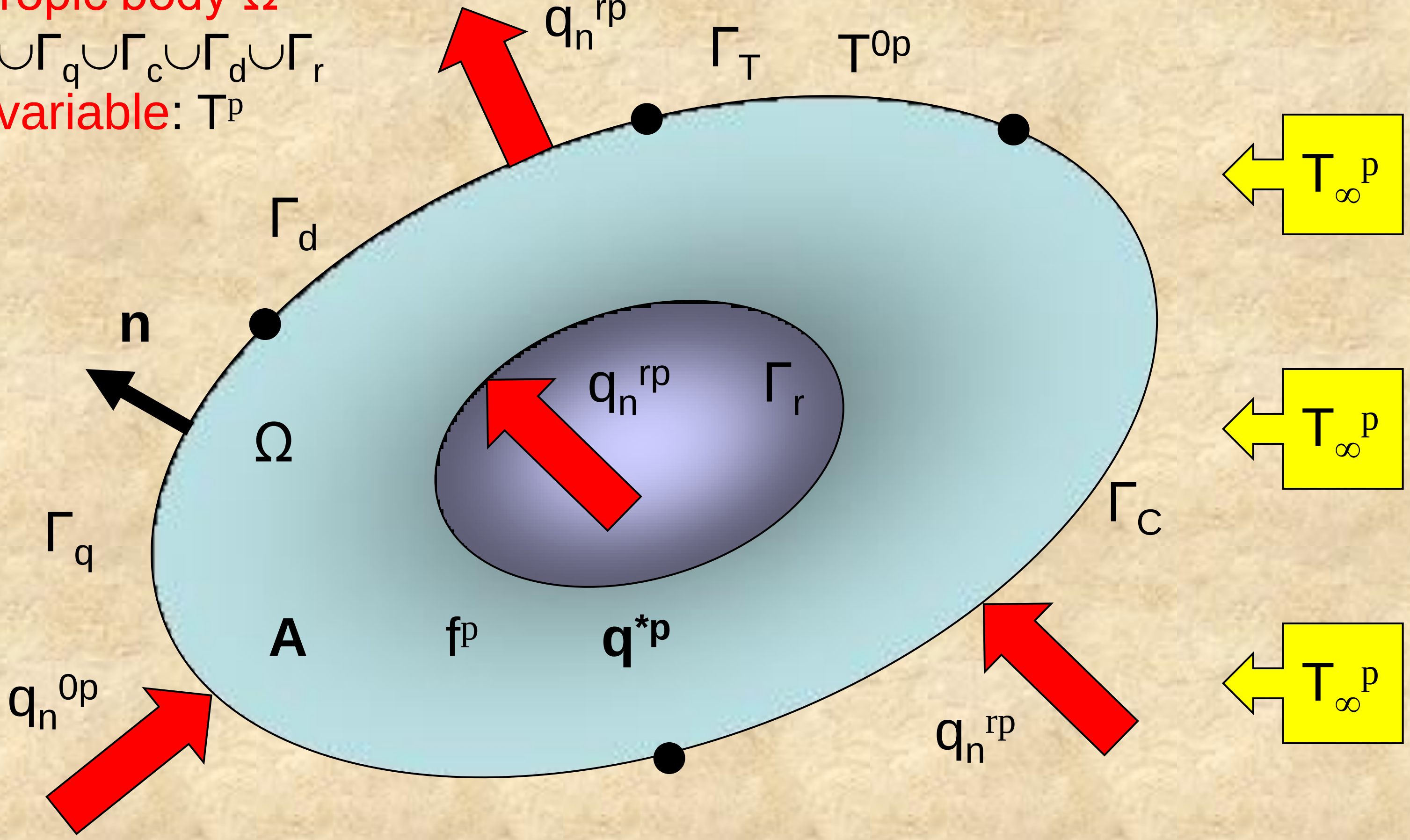
$F_p = DF/Db_p \quad p = 1...P$

DIRECT APPROACH

ADJOINT APPROACH

Sensitivity analysis. Direct approach

anisotropic body Ω
$\Gamma = \Gamma_T \cup \Gamma_q \cup \Gamma_c \cup \Gamma_d \cup \Gamma_r$
State variable: T^p



- The same: shapes, materials, heat conduction process.
- Additional fields within the domain and on the boundaries.

Sensitivity analysis. Direct approach

- The unknown fields = solutions of additional problems associated with each design parameter.
- Equations formulated by differentiation of equations for the primary problem.

Differential transport equation:

$$\left.\begin{aligned} -\operatorname{div}\mathbf{q}^p + f^p &= c\dot{T}^p \\ \mathbf{q}^p &= \mathbf{A}\cdot\nabla T^p + \mathbf{q}^{*p} \end{aligned}\right\} \text{within } \Omega; \quad \dot{T}^p = \frac{dT^p}{dt}$$

Boundary and initial conditions:

$$T^p(\mathbf{x},t) = T_p^0 - \nabla T^0 \cdot \mathbf{v}^p \ \text{on}\ \Gamma_T$$

$$q_n^p(\mathbf{x},t) = q_{np}^0 + \mathbf{q}_\Gamma^0 \cdot \nabla_\Gamma v_n^p - \nabla_\Gamma q_n^0 \cdot \mathbf{v}_\Gamma^p - q_{n,n}^0 v_n^p \ \text{on}\ \Gamma_q$$

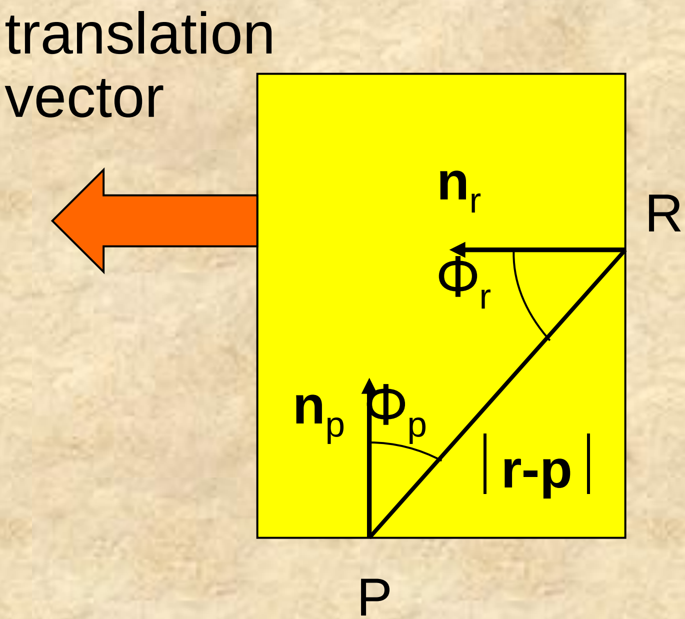
$$q_n^p(\mathbf{x},t) = h[T^p - T_\infty^p] + \mathbf{q}_\Gamma \cdot \nabla_\Gamma v_n^p \ \text{on}\ \Gamma_C$$

$$\mathbf{n}\cdot\mathbf{q}^{rp}(\mathbf{x},t) = q_n^{rp}(\mathbf{x},t) \ \text{on}\ \Gamma_r \ ;\ T_0^p(\mathbf{x},0) = T_{0p} - \nabla T_0 \cdot \mathbf{v}^p \ \text{on}\ \Omega\cup\Gamma$$

$$\left(q_n^r\right)_p = 4\sigma T^3\left(T^p + \nabla T\cdot\mathbf{v}^p\right) \quad \mathbf{x}\in\Gamma_d$$

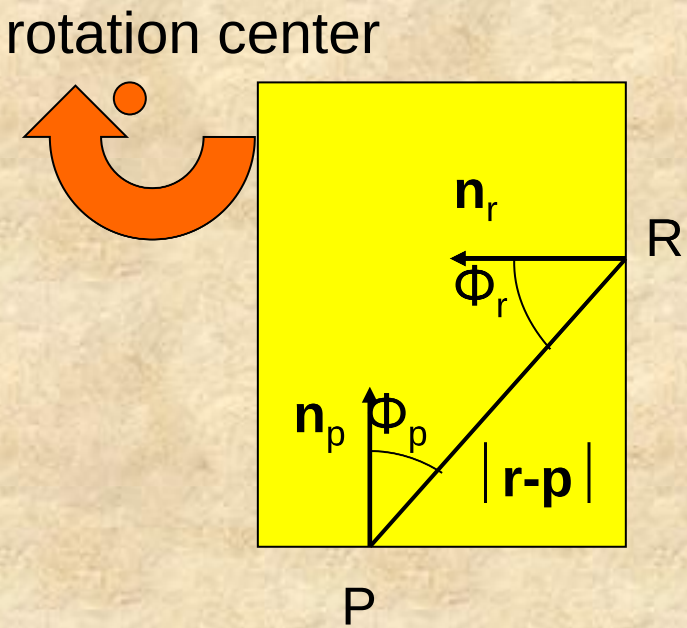
Sensitivity analysis. Direct approach. Radiation

Translation



The same: shape and dimensions
Modified: the location

Rotation

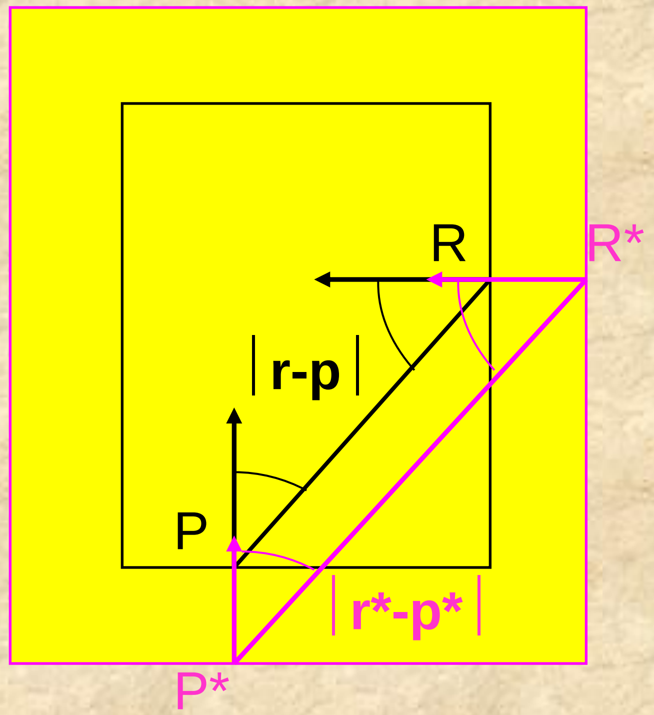


The same: shape and dimensions
Modified: the location

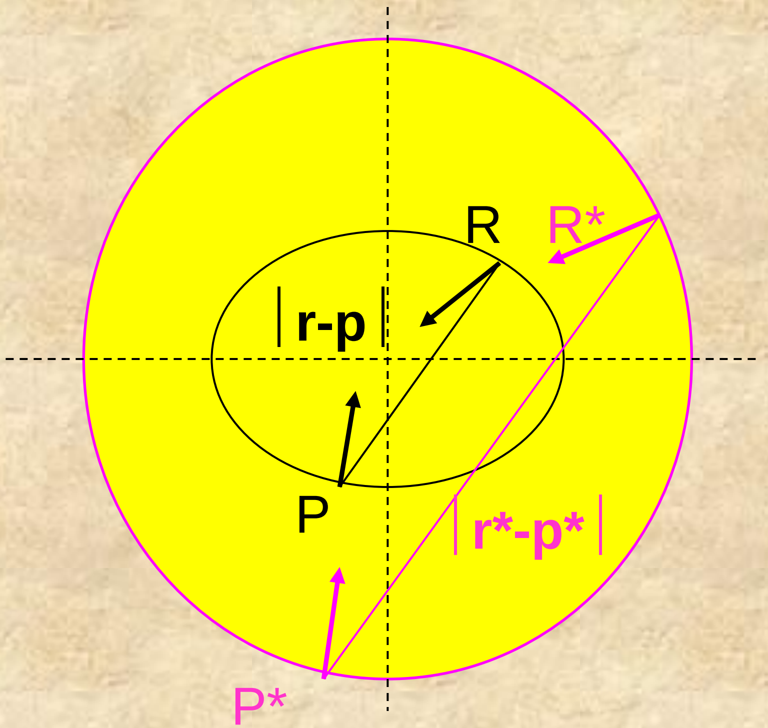
Kernel function is design parameter independent; $DK/Db_p=0$

Sensitivity analysis. Direct approach. Radiation

Expansion or contraction



Elliptical contour



The same: shape
Modified: the location and dimensions

The same: shape
Modified: the location and dimensions

Kernel function is design parameter dependent; $DK/Db_p \neq 0$

Sensitivity analysis. Direct approach. Radiation

$$
\begin{aligned}
&\left(q_n^r\right)^p - \mathbf{q}_\Gamma^r \cdot \nabla_\Gamma v_n^p + \nabla_\Gamma q_n^r \cdot \mathbf{v}_\Gamma^p + q_{n,n}^r v_n^p + 4\sigma T(\mathbf{p})^3\left[T^p(\mathbf{p}) + \nabla T(\mathbf{p})\cdot \mathbf{v}^p\right] = \\
&\varepsilon(\mathbf{p})\int_{\Gamma_r}\Big\{4\sigma\, T(\mathbf{r})^3\left[T^p(\mathbf{r}) + \nabla T(\mathbf{r})\cdot\mathbf{v}^p\right] + \\
&\frac{1-\varepsilon(\mathbf{r})}{\varepsilon(\mathbf{r})}\left[\left(q_n^r\right)^p - \mathbf{q}_\Gamma^r \cdot \nabla_\Gamma v_n^p + \nabla_\Gamma q_n^r \cdot \mathbf{v}_\Gamma^p + q_{n,n}^r v_n^p\right] K(\mathbf{r},\mathbf{p})\exp\left(-a|\mathbf{r}-\mathbf{p}|\right) d\Gamma_r + \\
&\varepsilon(\mathbf{p})\int_{\Gamma_r}\left\{\sigma\, T(\mathbf{r})^4 + \frac{1-\varepsilon(\mathbf{r})}{\varepsilon(\mathbf{r})} q_n^r(\mathbf{r})\right\} K(\mathbf{r},\mathbf{p})\exp\left(-a|\mathbf{r}-\mathbf{p}|\right)\left(\mathrm{div}_\Gamma \mathbf{v}^p - 2Hv_n^p\right) d\Gamma_r + \\
&4\,\varepsilon(\mathbf{p})\left(T^m\right)^p \sigma \left(T^m\right)^3 \int_{\Gamma_r} K(\mathbf{r},\mathbf{p})\exp\left(-a|\mathbf{r}-\mathbf{p}|\right) d\Gamma_r + \\
&\sigma\left(T^m\right)^4 \varepsilon(\mathbf{p})\int_{\Gamma_r} K(\mathbf{r},\mathbf{p})\exp\left(-a|\mathbf{r}-\mathbf{p}|\right)\left(\mathrm{div}_\Gamma \mathbf{v}^p - 2Hv_n^p\right) d\Gamma_r
\end{aligned}
$$

- radiation properties are position independent $D\varepsilon/Db_p = D\varepsilon/Db_r = 0$; translation and rotation.

Sensitivity analysis. Direct approach. Correlation

$$
\begin{aligned}
F_P = & \left[\int_\Omega \Psi,_{\dot{T}}\, T^p d\Omega\right]_0^{t_f} + \int_0^{t_f}\Bigg\{\int_\Omega\left[\left(\Psi,_T - \frac{d(\Psi,_{\dot{T}})}{dt}\right)T^p + \nabla_{\nabla T}\Psi\cdot\nabla T^p + \nabla_{\mathbf{q}}\Psi\cdot\mathbf{q}^p + \Psi,_f\, f^p\right]d\Omega + \\
& \int_{\Gamma_T}\left[\gamma,_T\left(T_p^0 - \nabla_\Gamma T^0\cdot\mathbf{v}_\Gamma^p - T,_n^0\, v_n^p\right) + \gamma,_{q_n}\left(q_n^p - \mathbf{q}_\Gamma\cdot\nabla_\Gamma v_n^p\right)\right]d\Gamma_T + \\
& \int_{\Gamma_q}\left[\gamma,_T\, T^p + \gamma,_{q_n}\left(q_{np}^0 - \nabla_\Gamma q_n^0\cdot\mathbf{v}_\Gamma^p - q_n^0,_n\cdot v_n^p\right)\right]d\Gamma_q + \int_{\Gamma_c}\left[\gamma,_T\, T^p + \gamma,_{q_n} h\left(T^p - T_\infty^p\right)\right]d\Gamma_c + \\
& \int_{\Gamma_d}\gamma,_T\, T^p d\Gamma_d + \int_\Gamma\left(\Psi + \gamma,_n - 2H\gamma\right)v_n^p\, d\Gamma + \int_\Gamma \gamma,_{T_\infty} T_\infty^p\, d\Gamma + \int_\Sigma \left]\gamma\mathbf{v}^p\cdot\boldsymbol{\upsilon}\right[\Bigg\}dt
\end{aligned}
$$

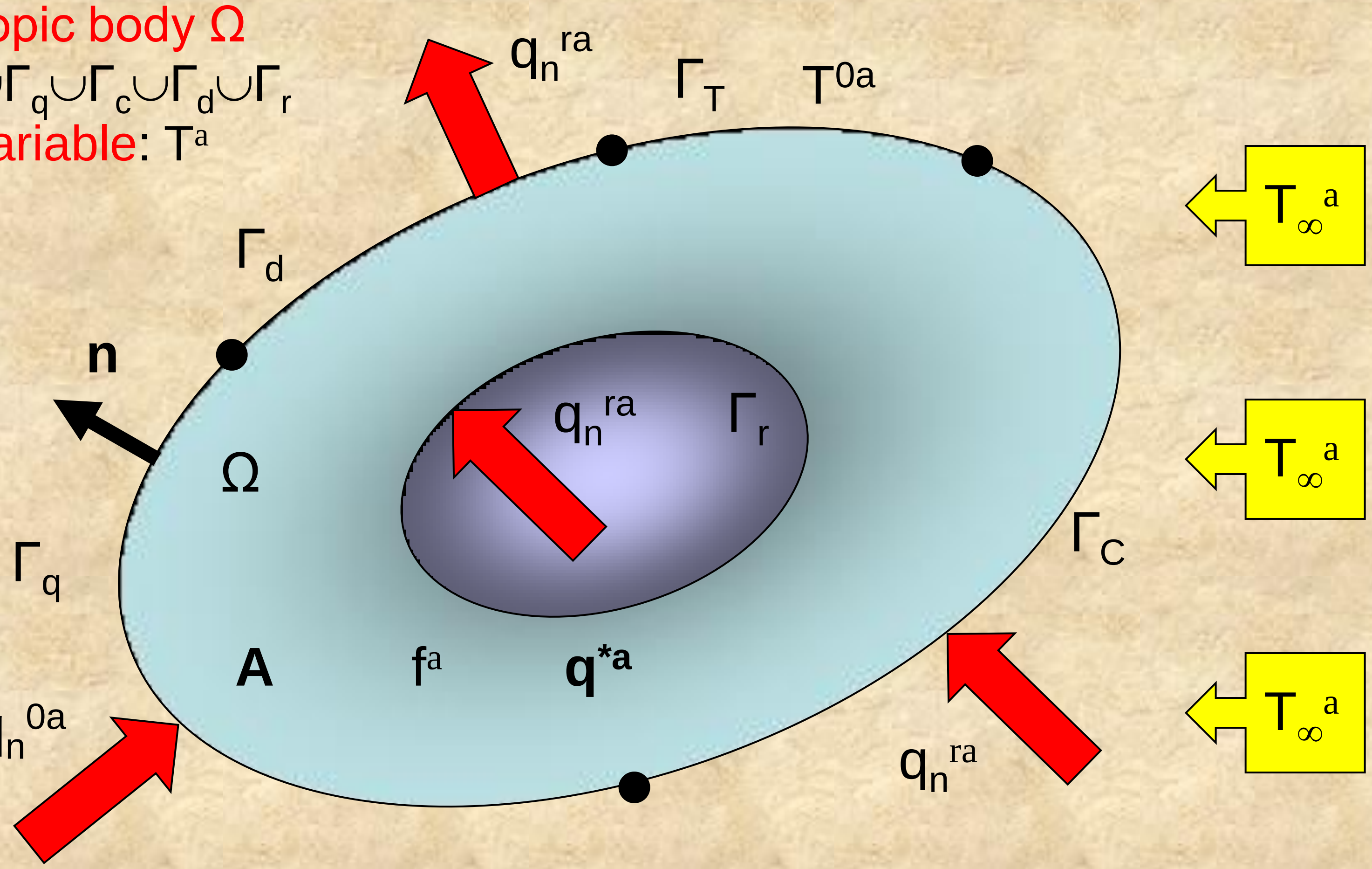
- **P** design variables + **N** objective functionals = **1** primary problem + **P** additional problems
- Convenient for the simple geometry of structure and the multicriterial optimization (considerable number of **N**)

Sensitivity analysis. Adjoint approach

anisotropic body Ω

$\Gamma = \Gamma_T \cup \Gamma_q \cup \Gamma_c \cup \Gamma_d \cup \Gamma_r$

State variable: T^a



- The same: shapes, materials, heat conduction process
- Additional fields within the domain and on the boundary

Sensitivity analysis. Adjoint approach

- The unknown fields = solutions of adjoint problems

Differential transport equation:

$$\left.\begin{aligned} -\operatorname{div}\mathbf{q}^a + f^a &= c\dot{T}^a \\ \mathbf{q}^a &= \mathbf{A}\cdot\nabla T^a + \mathbf{q}^{*a} \end{aligned}\right\} \text{in } \Omega;\ \dot{T}^a = \frac{dT^a}{d\tau} = -\frac{dT^a}{dt}$$

Boundary and initial conditions:

$$T^a(\mathbf{x},\tau) = T^{0a}(\mathbf{x},\tau)\ \mathbf{x}\in\Gamma_T \quad ; \quad q_n^a(\mathbf{x},\tau) = \mathbf{n}\cdot\mathbf{q}^a = q_n^{0a}(\mathbf{x},\tau)\ \mathbf{x}\in\Gamma_q$$

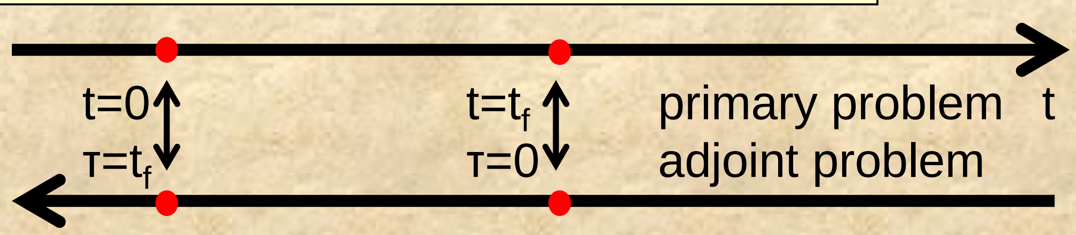
$$q_n^a(\mathbf{x},\tau) = \mathbf{n}\cdot\mathbf{q}^a = h\left[T^a(\mathbf{x},\tau) - T_\infty^a(\mathbf{x},\tau)\right]\ \mathbf{x}\in\Gamma_c$$

$$\mathbf{n}\cdot\mathbf{q}^{ar} = q_n^{ar}(\mathbf{x},\tau)\quad \mathbf{x}\in\Gamma_r \qquad\qquad \mathbf{n}\cdot\mathbf{q}^{ar} = q_n^{ar}(\mathbf{x},\tau)\quad \mathbf{x}\in\Gamma_d$$

$$T^a(\mathbf{x},\tau=0) = T_0^a(\mathbf{x},\tau=0)\qquad \mathbf{x}\in(\Omega\cup\Gamma)$$

Time transformation

τ



Sensitivity analysis. Adjoint approach

- Volume and boundary conditions = by means of the heat transport equation

$$T^a(\mathbf{x},\tau=0)=\frac{1}{c}\Psi,_{\dot{T}}(\mathbf{x},t=t_f) \qquad \mathbf{x}\in(\Omega\cup\Gamma)$$

$$f^a(\mathbf{x},\tau)=\Psi,_T(\mathbf{x},t)-\frac{d}{dt}\Psi,_{\dot{T}}(\mathbf{x},t) \quad \mathbf{x}\in\Omega$$

$$\mathbf{q}^{*a}(\mathbf{x},\tau)=\nabla_{\nabla T}\Psi(\mathbf{x},t)+\nabla_{\mathbf{q}}\Psi(\mathbf{x},t)\cdot\mathbf{A}(\mathbf{x}) \quad \mathbf{x}\in\Omega$$

$$T^{0a}(\mathbf{x},\tau)=\gamma,_{q_n}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_T \qquad q_n^{0a}(\mathbf{x},\tau)=-\gamma,_T(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_q$$

$$T_\infty^a(\mathbf{x},\tau)=\frac{1}{h}\gamma,_T(\mathbf{x},t)+\gamma,_{q_n}(\mathbf{x},t) \qquad \mathbf{x}\in\Gamma_c$$

$$q_n^{ar}(\mathbf{x},\tau)=\sigma\left[T^a(\mathbf{x},\tau)\right]^4;\ T^a(\mathbf{x},\tau)=\left[\frac{-\gamma,_T(\mathbf{x},t)}{\sigma}\right]^{0,25} \qquad \mathbf{x}\in\Gamma_d$$

- Radiation condition = similar to the form of primary struct.

Sensitivity analysis. Adjoint approach. Correlation

$$F_p = -\left[\int_\Omega \left(\Psi,_{\dot{T}} - cT^a\right)\left(T_p - \nabla T \cdot \mathbf{v}^p\right) d\Omega\right]_{t=0} + \int_0^{t_f} \left\{ \int_\Omega \left[\left(\nabla_q \Psi + \nabla T^a\right) \cdot \mathbf{q}^{*p} + \left(\Psi,_f + T^a\right) f^p\right] d\Omega + \right.$$

$$\int_{\Gamma_T} \left[\left(\gamma,_T + q_n^a\right)\left(T_p^0 - \nabla_\Gamma T^0 \cdot \mathbf{v}_\Gamma^p - T,_n^0 v_n^p\right) - \gamma,_{q_n} \mathbf{q}_\Gamma \cdot \nabla_\Gamma v_n^p\right] d\Gamma_T +$$

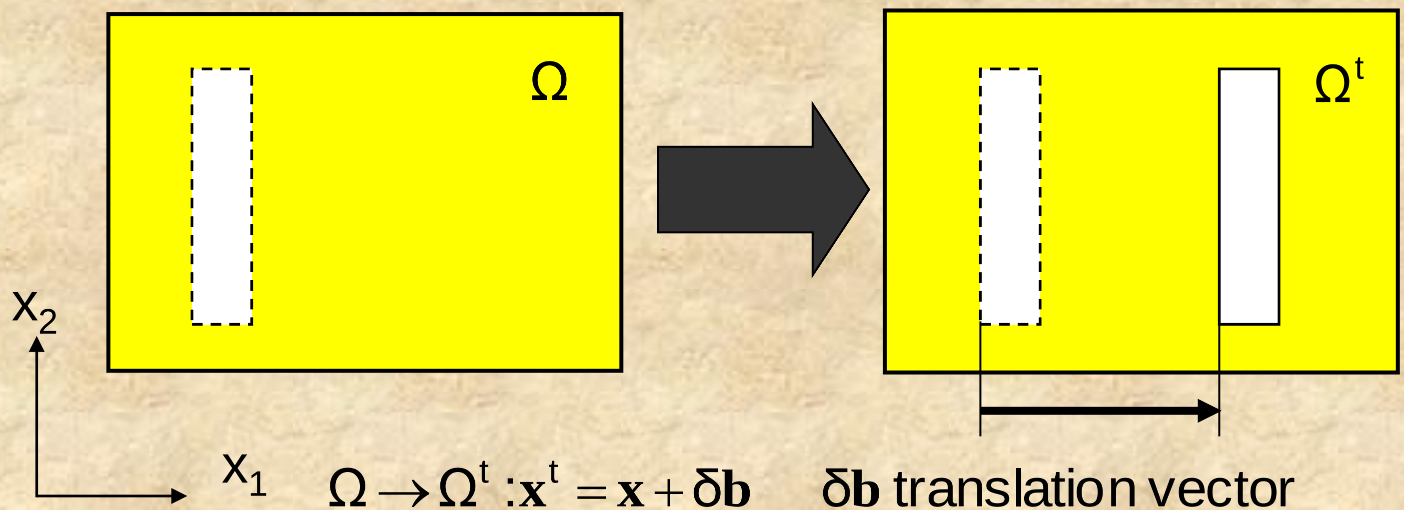
$$\int_{\Gamma_q} \left[\left(\gamma,_{q_n} - T^a\right)\left(q_{np}^0 - \nabla_\Gamma q_n^0 \cdot \mathbf{v}_\Gamma^p - q_n^0,_n v_n^p\right) - T^a \mathbf{q}_\Gamma^0 \cdot \nabla_\Gamma v_n^p\right] d\Gamma_q +$$

$$\int_{\Gamma_c} \left[T^a h T_\infty^p - T^a \mathbf{q}_\Gamma \cdot \nabla_\Gamma v_n^p - \gamma,_{q_n} h T_\infty^p\right] d\Gamma_c - \int_{\Gamma_d} T^a q_n^{rp}\, d\Gamma_d + \int_\Gamma \left(\Psi + \gamma,_n - 2H\gamma\right) v_n^p\, d\Gamma$$

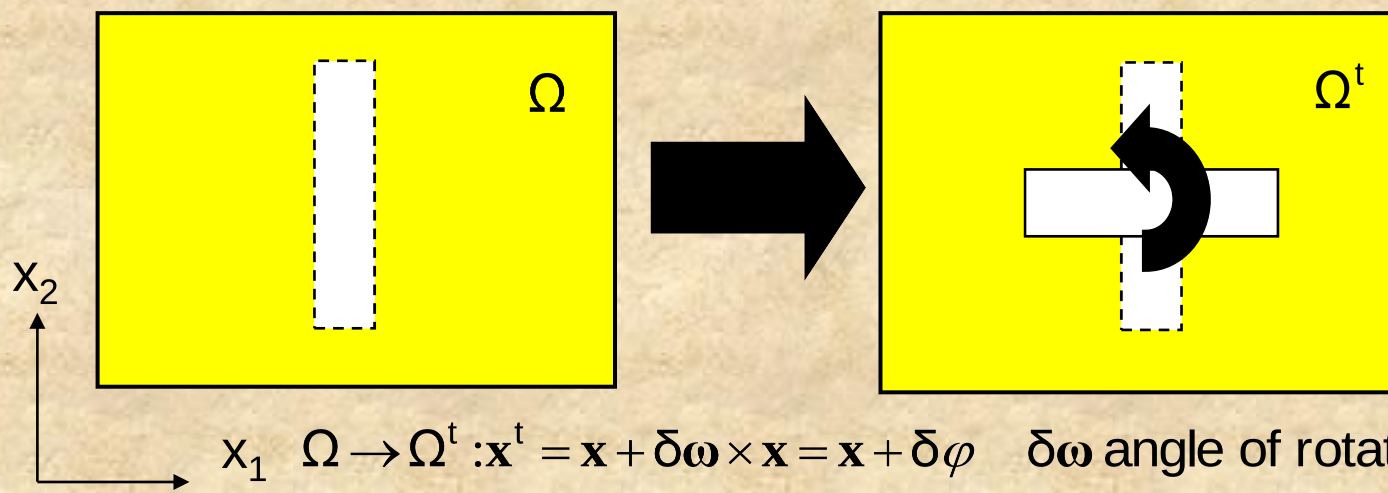
$$\left. + \int_\Gamma \gamma,_{T_\infty} T_\infty^p\, d\Gamma + \int_\Sigma \left]\gamma \mathbf{v}^p \cdot \boldsymbol{\upsilon}\right[\right\} dt$$

- **P** design variables + **N** objective functionals =
  1 primary problem + **N** adjoint problems
- Convenient for the complicated geometry of the structure and the one objective functional

Methods of shape modification

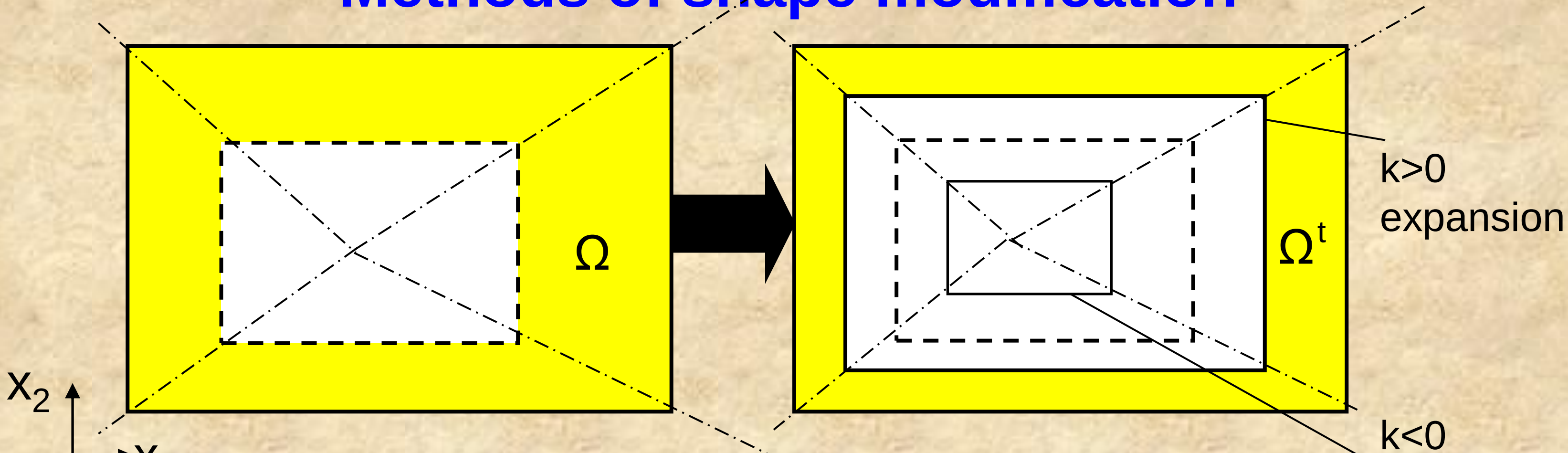


$\Omega \rightarrow \Omega^t : \mathbf{x}^t = \mathbf{x} + \delta\mathbf{b}$ $\delta\mathbf{b}$ translation vector

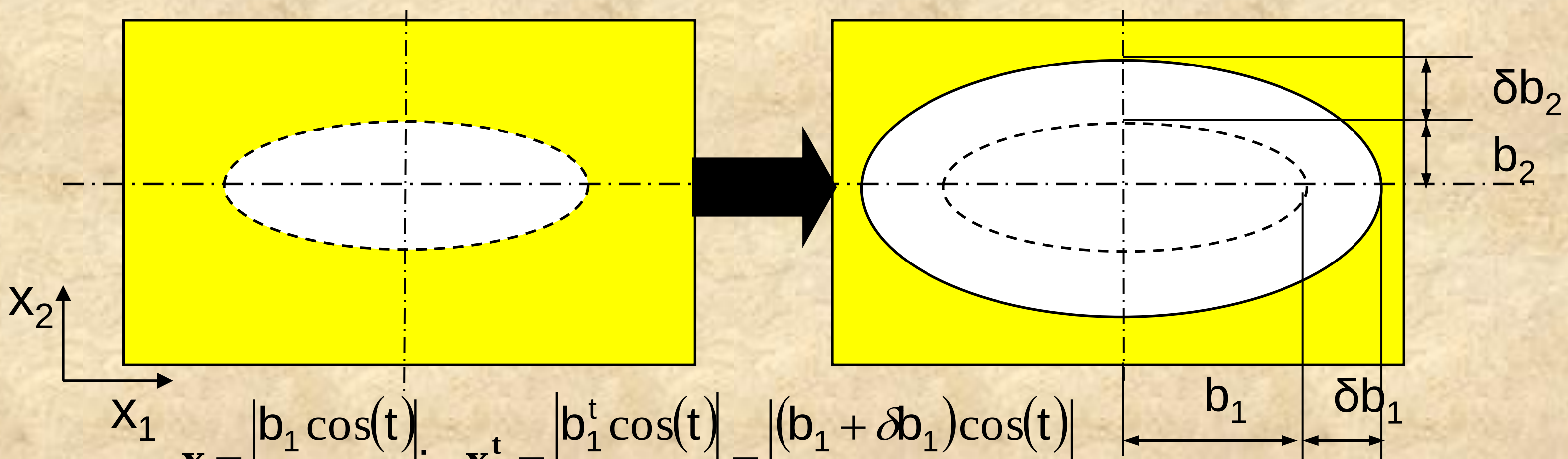


$\Omega \rightarrow \Omega^t : \mathbf{x}^t = \mathbf{x} + \delta\boldsymbol{\omega} \times \mathbf{x} = \mathbf{x} + \delta\varphi$ $\delta\boldsymbol{\omega}$ angle of rotation

Methods of shape modification



$\Omega \to \Omega^t : \mathbf{x}^t = \mathbf{x} + \delta k\, \mathbf{x};$ k scale change parameter



$$\mathbf{x} = \begin{vmatrix} b_1 \cos(t) \\ b_2 \sin(t) \end{vmatrix}; \quad \mathbf{x}^t = \begin{vmatrix} b_1^t \cos(t) \\ b_2^t \sin(t) \end{vmatrix} = \begin{vmatrix} (b_1 + \delta b_1)\cos(t) \\ (b_2 + \delta b_2)\sin(t) \end{vmatrix}$$

Shape optimization problem

Creation of a new shape of the textile structure

Optimality conditions

$$\begin{cases} \min\ G \\ C - C_0 = \int\limits_\Omega c_w \, d\Omega - C_0 = 0 \end{cases} \quad \text{or} \quad \begin{cases} \max\ G = \min\ (-G) \\ C - C_0 = \int\limits_\Omega c_w \, d\Omega - C_0 = 0, \end{cases}$$

G, C   optimization/constraint functionals,
C_0 total cost of the material, assumed as constant,
c_w unit material cost.

Introducing the Lagrange functional = optimality conditions

$$\begin{cases} \dfrac{DG}{Db_p} = -\chi \dfrac{DC}{Db_p} = -\chi \int\limits_\Omega c_w v_n^p d\Gamma \\ \int\limits_\Omega c_w \, d\Omega - C_0 = 0. \end{cases}$$

DG/Db_p; DC/Db_p 1st order sensitivities of the optimization/constraint functionals with respect to design parameter b_p,
χ Lagrange multiplier.

Shape optimization problem. Functionals

- Heat flux density normal to the external boundary

$$G = \int_0^{t_f} \left[\int_\Gamma q_n d\Gamma \right] dt;$$

Heat radiator = Maximization of the functional
Heat isolator = Minimization of the functional

- Measure of the temperature

$$G = \int_0^{t_f} \left\{ \left[\int_\Gamma \left(\frac{T}{T_0} \right)^n d\Gamma \right]^{\frac{1}{n}} \right\} dt \ ; \ n \to \infty;$$

T_0 assumed temperature level

Minimization of the functional =
- Equalization of the temperature distribution
- Minimization of the maximal local temperatures

Shape optimization problem. Functionals

- Amount of heat generated within domain

$$G = \int_0^{t_f} \left[\int_\Omega f \, d\Omega \right] dt.$$

Heat radiator =	Maximization of the functional
Heat isolator =	Minimization of the functional

- Global behavioral measure of structural thermal response

Increase of structural entropy – the entropy formulation according to Gyarmati

$$\Pi_T = \int_0^{t_f} \left[\int_\Omega \left(\frac{1}{2} \nabla T \cdot \mathbf{A} \cdot \nabla T + fT \right) d\Omega - \int_{\Gamma_c} h \left(\frac{1}{2} T^2 - T T_\infty \right) d\Gamma_c - \int_{\Gamma_q} q_n^0 T \, d\Gamma_q \right] dt$$

Minimization of the objective functional = maximization of the outcoming entropy

Shape identification problem

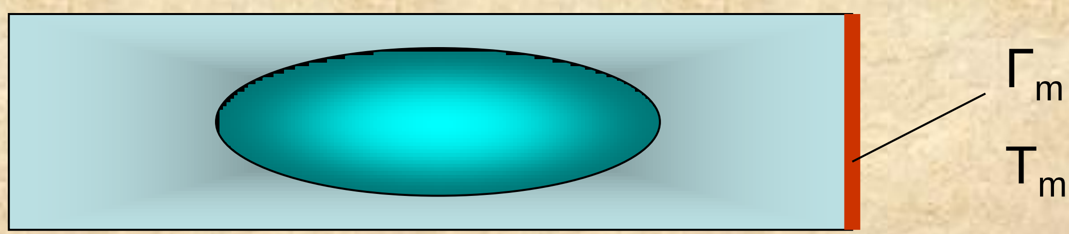
- Determination of the real shape by means of the temperature for the current model and the real body,
- Minimization of the objective functional without constraints

Stationarity conditions:

$$\min\ G \Rightarrow G_p = \frac{DG}{Db_p} = 0$$

DG/Db_p 1[st] order sensitivity of the identification functional,
b_p set of design parameters ($p = 1 \ldots P$),
P number of design parameters.

Temperature measured on part Γ_m of external boundary



Shape identification problem

- "Distance" between the temperatures T of the identified model and T_m of the real product measured on the part Γ_m:

$$G = \frac{1}{2}\int_0^{t_f}\left[\int_{\Gamma_m}\left(T - T_m\right)^2 d\Gamma_m\right]dt$$

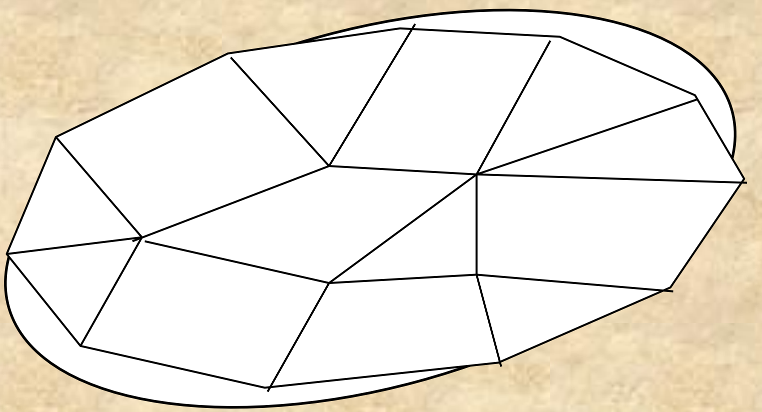
- Measure of the temperature:
  - Functional is homogeneous,
  - Suitable for expansion/contraction of the boundary,
  - Minimization of G reduces the „distance" between T and T_m,
  - Minimization of G minimizes the maximal local temperatures.

$$G = \int_0^{t_f}\left\{\left[\int_{\Gamma_m}\left(\frac{T}{T_m}\right)^n d\Gamma_m\right]^{\frac{1}{n}}\right\}dt \;;\; n \to \infty$$

- Acc. Damage Location Assurance Criterion, introduced by Messina, Krawczuk, Żak, Ostachowicz,
  - Range of correlations: 0 (no correlation) – 1 (full correlation)

$$G = \int_0^{t_f}\left[\frac{\left(\int_{\Gamma_m} T_m T \, d\Gamma_m\right)^2}{\int_{\Gamma_m} T_m T_m \, d\Gamma_m \int_{\Gamma_m} T\, T \, d\Gamma_m}\right]dt$$

Numerical methods of heat transfer solution



- Finite Element Method FEM

Advantages	Disadvantages
- T determined within the domain, - Contains the combined linear and nonlinear elements, - Good approximation of the shape, - Easy for the linear boundary, - The independent interpolation of the domain and the objective functional, - Many commercial and processing programs.	- The whole domain should be discretized, - The considerable long calculation time, - Difficult to numerical calculations for the large domains.

Numerical methods of heat transfer solution

FEM contains 3 approaches:

- Variational approach: minimization of the functional of the clear physical interpretation, i.e. solution of the differential description,

Advantages	Disadvantages
- The direct physical interpretation.	- The physical interpretation not always existing or unclear.

- Galerkin’s approach: residuum defined as the integral of the temperature field incompability, decomposed within the domain,

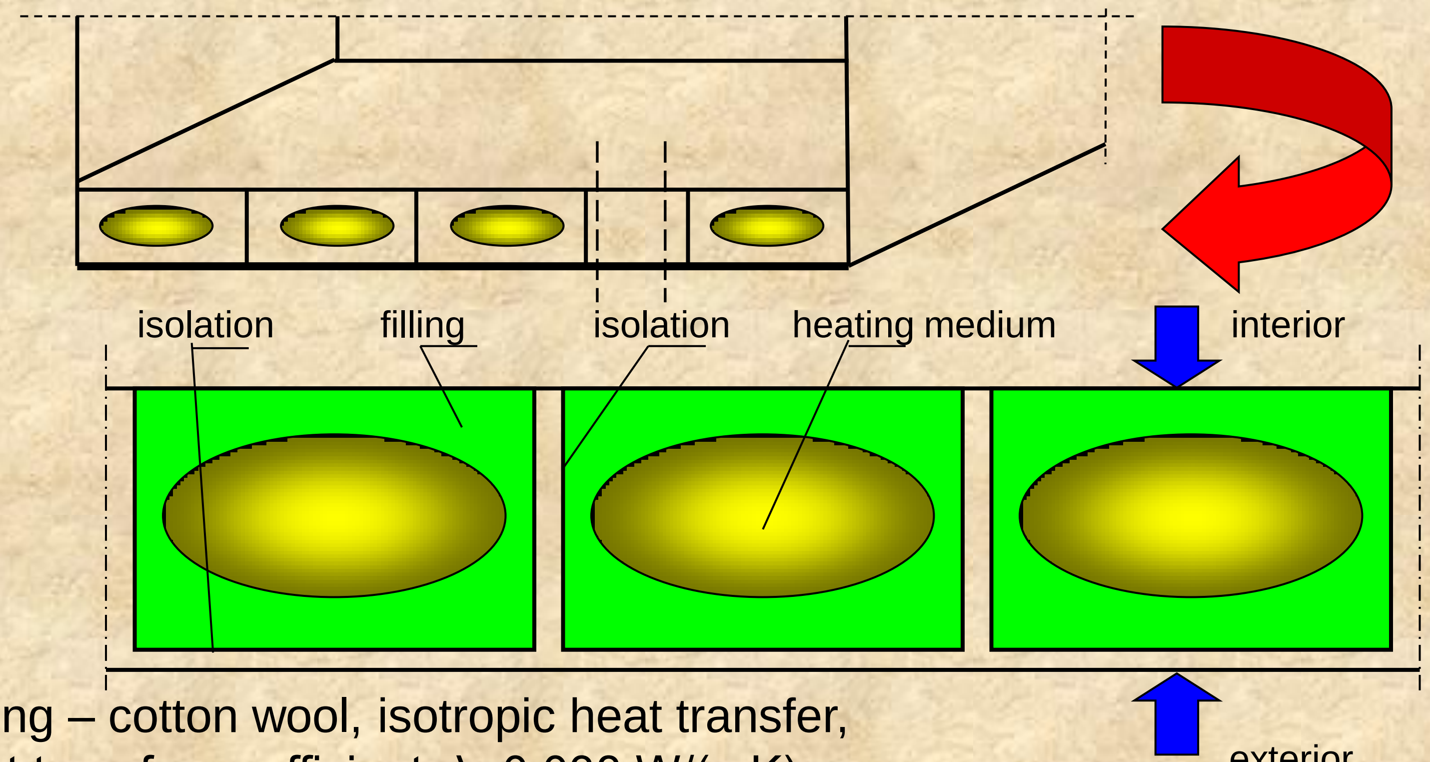
- Can be always applied.	- The physical interpretation not always existing or unclear.

- Balance approach: heat balances formulated within the control approaches,

- Continous heat within the control domains.	- The control approaches are formulated irrespective of the FE net.

Shape optimization/identification in textile structures

Shape identification of heating channel within heating floor pannels during transient heat transport



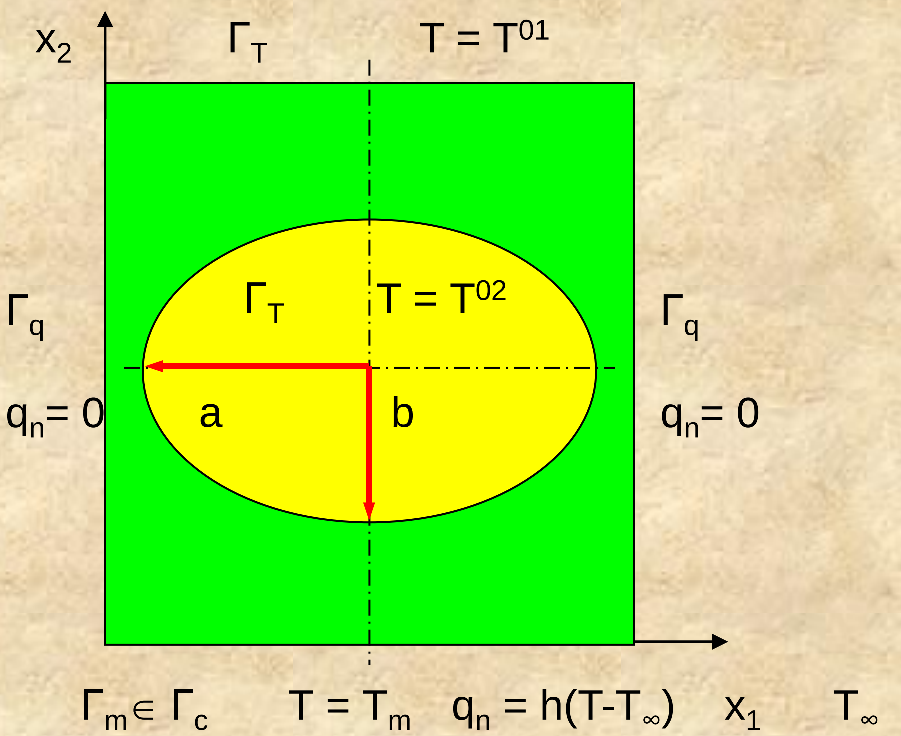
- Filling – cotton wool, isotropic heat transfer, heat transfer coefficient λ=0,038 W/(mK)
- Hole has probably the eliptic shape/no access inside. Shape modification: lengths of the semi-axes (2 identification parameters)

Technical University of Lodz
Department of Technical Mechanics and Informatics

Shape identification of heating channel within heating floor pannels during transient heat transport

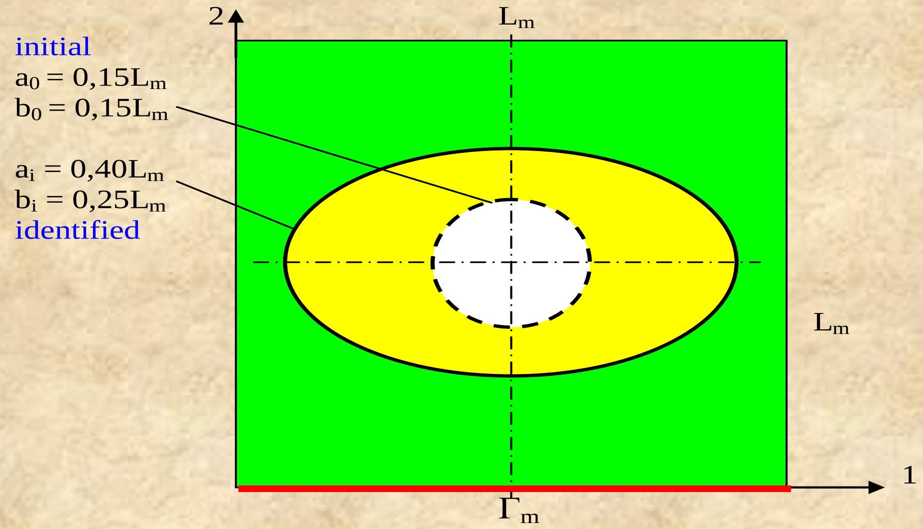
Identification functional

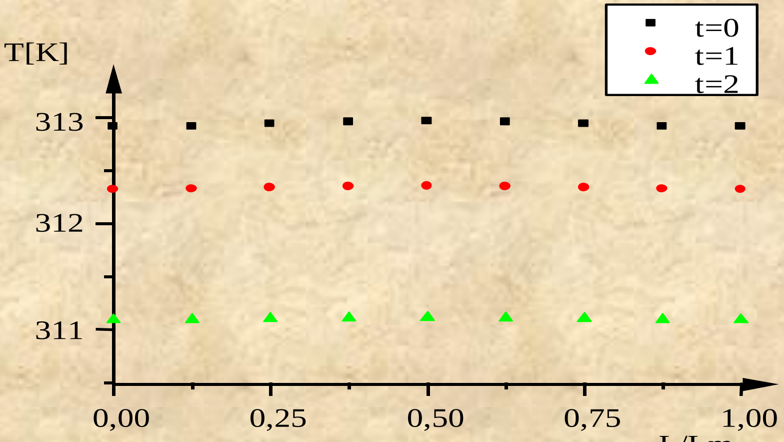
$$G = \frac{1}{2}\int_0^{t_f}\left[\int_{\Gamma_m}(T - T_m)^2 d\Gamma_m\right]dt \rightarrow \min$$



- Side boundaries: isolation
- Lower boundary: convection, measured temp. $\Gamma_m \in \Gamma_c$; T_∞=280K
- Hole – heating medium – assumed temperature in time $T = 313\ (2{,}718)^{-0{,}002t}$ - [K], $t_{pocz}=0$; $t_{konc}=7$s, $\Delta t=1$s
- Interior – assumed temperature $T = 293 + 10 \sin(\frac{1}{4}\Pi t)$ [K], $t_{pocz}=0$; $t_{konc}=7$s, $\Delta t=1$s
- Analysis stage: FE net of 4-nodal elements, direct approach to sensitivity analysis
- Synthesis stage: external penalty function.

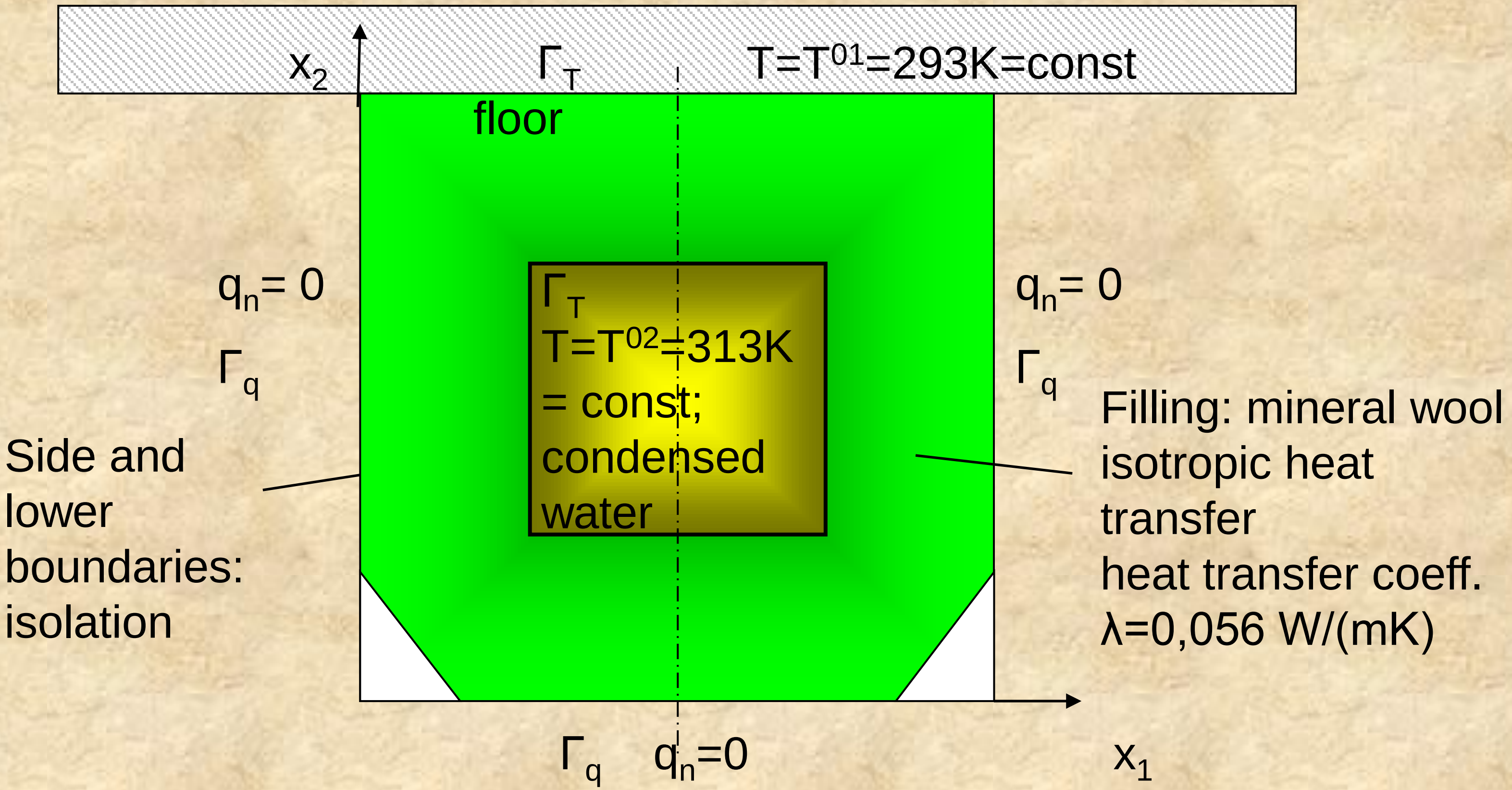
Shape identification of heating channel within heating floor pannels during transient heat transport





Temperature distribution: boundary Γ_m time t = (0, 1, 2) s

Shape optimization of outlet pipe transporting condensed water vapor from dryer. Transient heat transfer



Shape optimization of outlet pipe transporting condensed water vapor from dryer. Transient heat transfer

Optimization problem

$$G = \int_{\Gamma_T} q_n \, d\Gamma_T \rightarrow \min \quad ; \quad C = C_0$$

- Let us introduce the quadrangular shape of the hole.
  Design variables: the coordinates of the vertexes,
- Analysis stage: FE net of 4-nodal elements, direct and adjoint approaches to sensitivity analysis
- Synthesis stage: external penalty function.

Primary problem

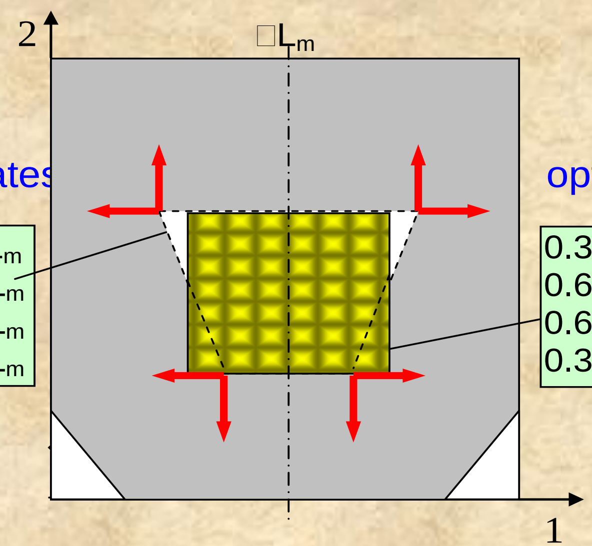
$$\mathrm{div}\,\mathbf{q} = 0 \ \text{ within } \Omega; \quad \mathbf{q} = \mathbf{A} \cdot \nabla T \quad \text{within } \Omega$$

$$T = T^0 \text{ on } \Gamma_T \quad ; \quad q_n = 0 \text{ on } \Gamma_q \quad ; \quad q_n = h\left(T - T_\infty\right) \text{ on } \Gamma_c$$

Shape optimization of outlet pipe transporting condensed water vapor from dryer. Transient heat transfer

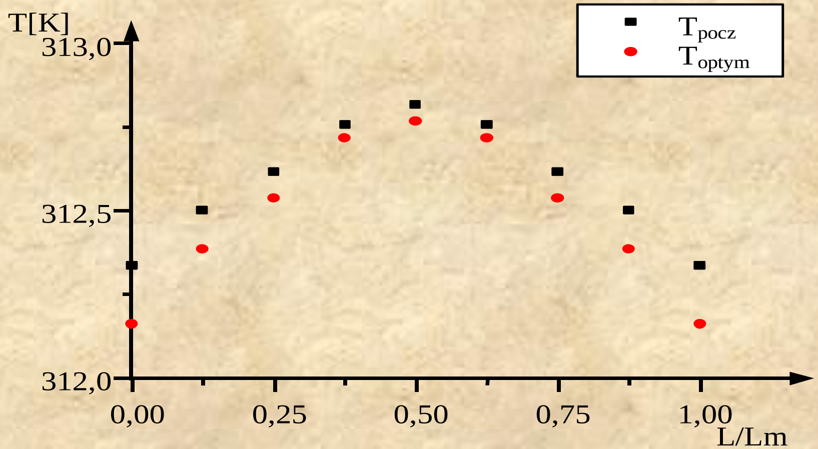
initial coordinates

$0.350L_m$	$0.250L_m$
$0.650L_m$	$0.250L_m$
$0.800L_m$	$0.650L_m$
$0.200L_m$	$0.650L_m$



optimal coordinates

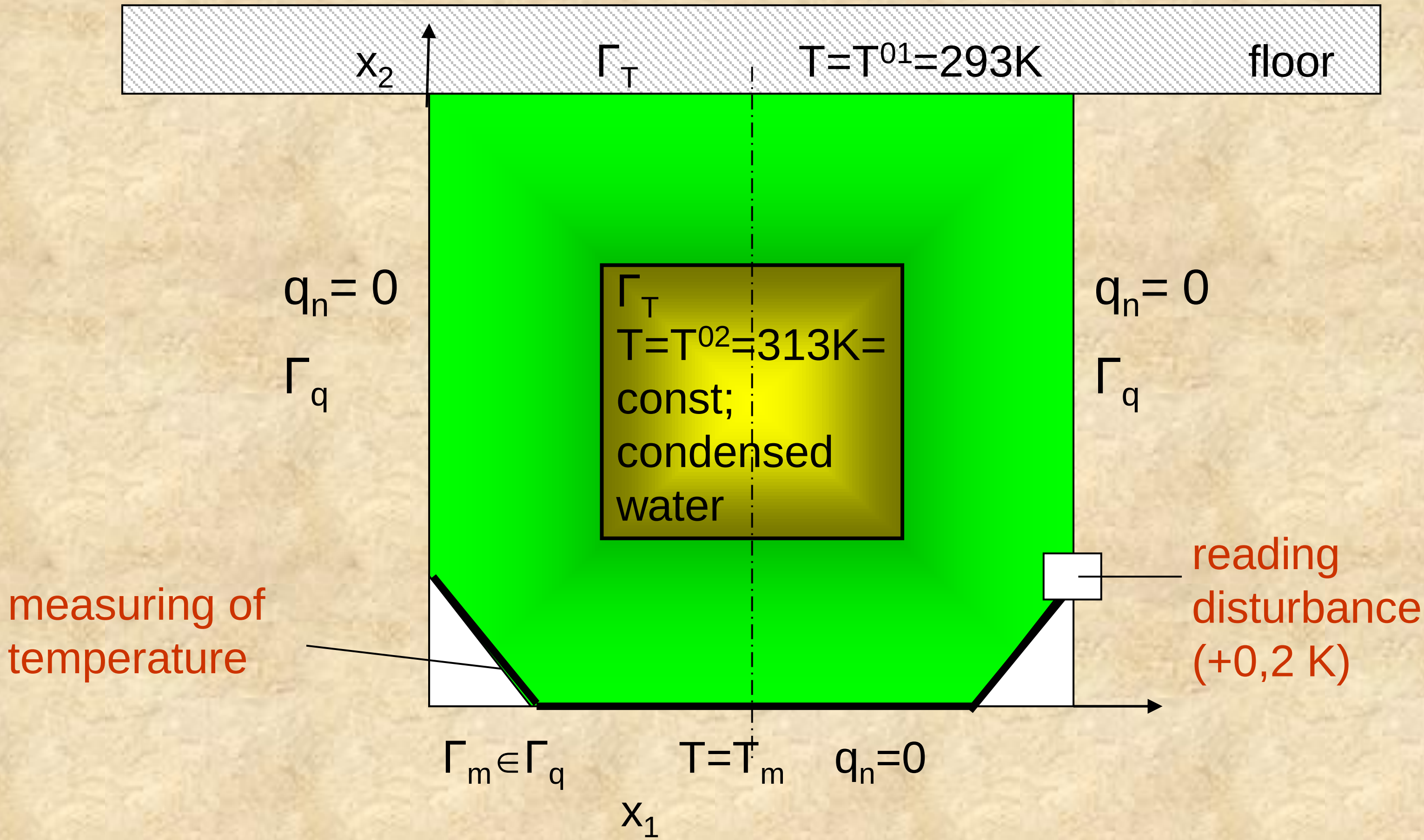
$0.313L_m$	$0.254L_m$
$0.688L_m$	$0.254L_m$
$0.695L_m$	$0.653L_m$
$0.306L_m$	$0.653L_m$



Temperatures: initial (T_{pocz}) and optimal shapes (T_{optym}) on lower bound. Γ_q

Objective functional decreases about 18,34 %

Shape identification of outlet pipe transporting condensed water vapor from dryer. Transient heat transfer



Shape identification of outlet pipe transporting condensed water vapor from dryer. Transient heat transfer

- Identification of the above optimal hole
(i.e. the quadrangular shape of the optimal location of vertexes)
- Hole can be translated vertically (cf. the fabrication errors),
shape modification: vertical translation - 1 identification param.,
- Boundary $\Gamma_m \in \Gamma_q$: measuring of temperature
(the pointwise reading disturbance +0,2K)
- Identification functionals:

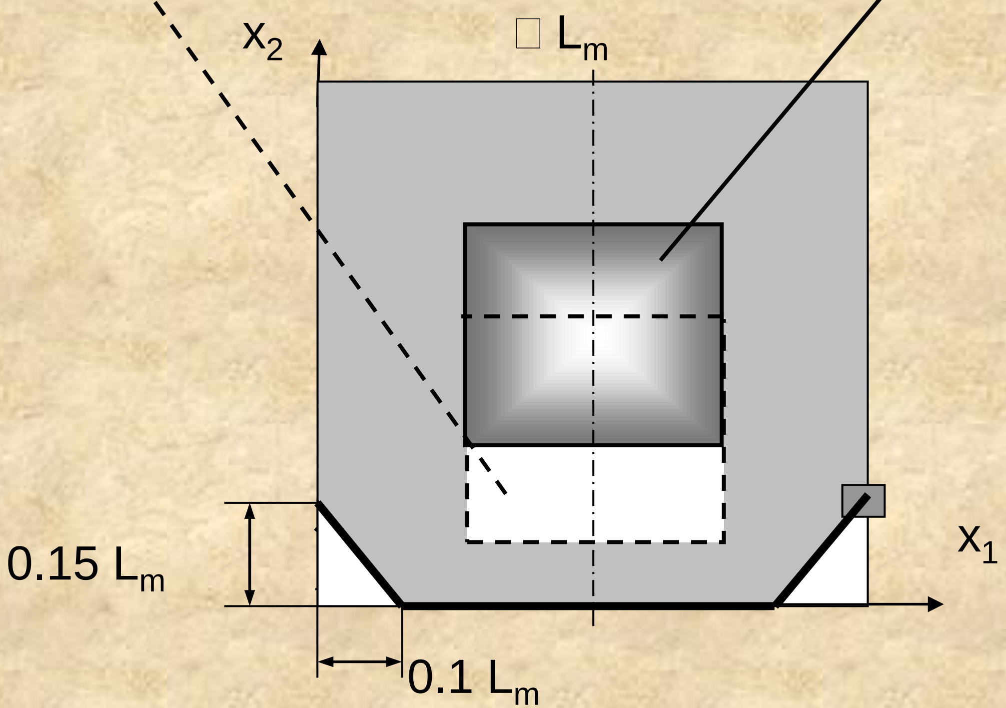
$$G = \frac{1}{2}\int_{\Gamma_m} \left(T - T_m\right)^2 d\Gamma_m \rightarrow \min \qquad G = \frac{\left(\int_{\Gamma_m} T_m T \, d\Gamma_m\right)^2}{\int_{\Gamma_m} T_m T_m \, d\Gamma_m \int_{\Gamma_m} T\,T \, d\Gamma_m} \rightarrow \min$$

- Analysis stage: FE net of 4-nodal elements, direct approach to sensitivity analysis.
- Synthesis stage: external penalty function.

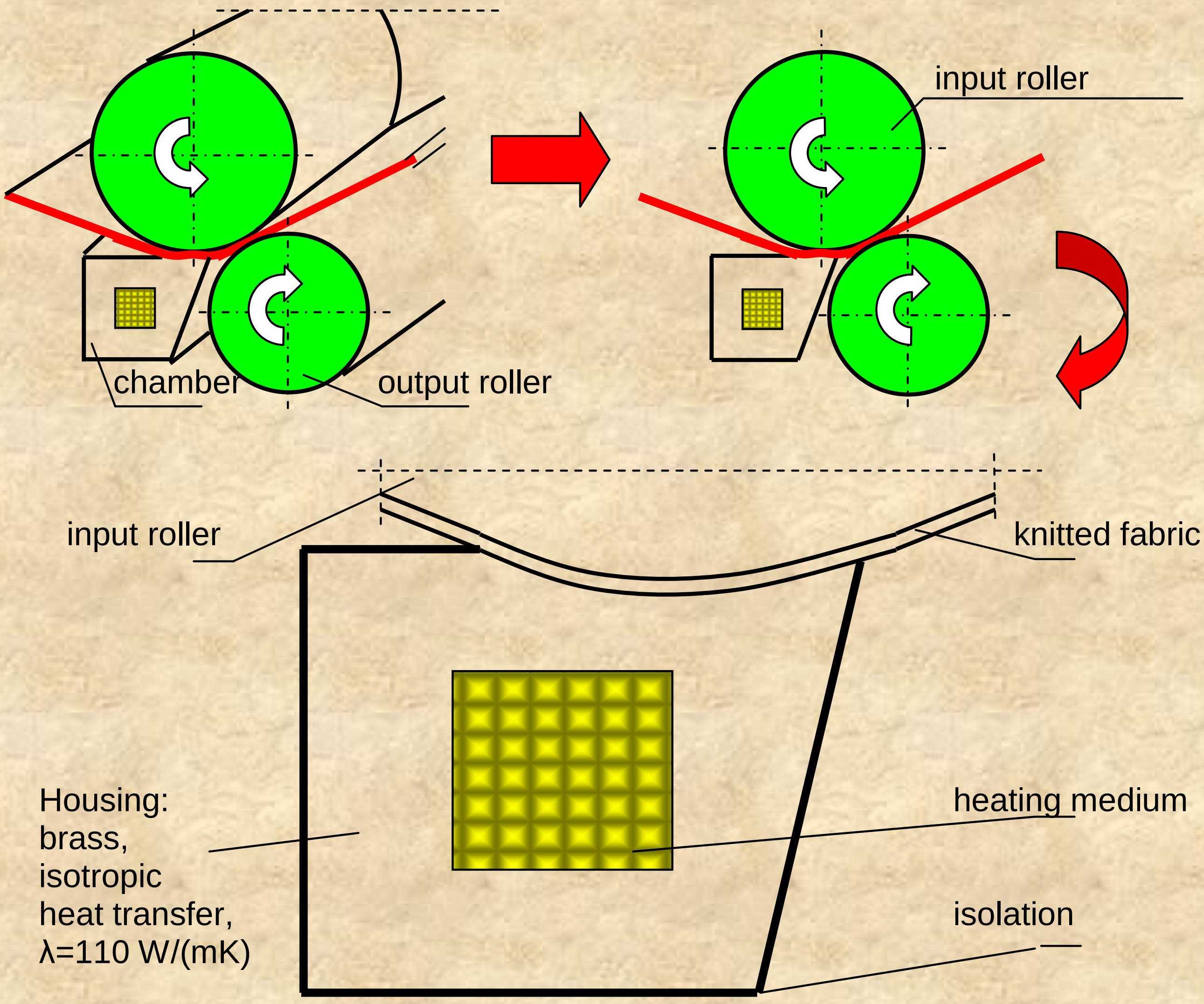
Shape identification of outlet pipe transporting condensed water vapor from dryer. Transient heat transfer

initial coordinates	
$0,290L_m$	$0,144L_m$
$0,711L_m$	$0,144L_m$
$0,723L_m$	$0,543L_m$
$0,276L_m$	$0,543L_m$

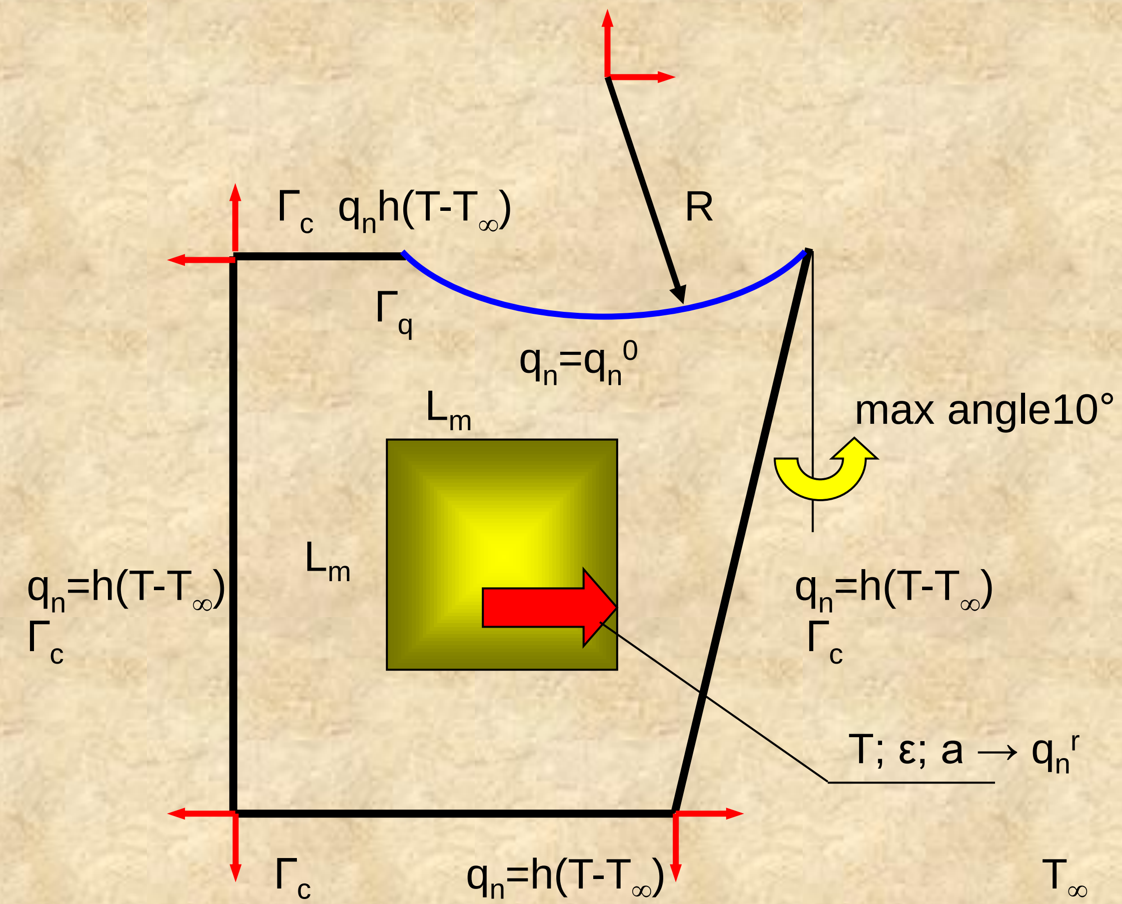
identified corrdinates	
$0,290L_m$	$0,264L_m$
$0,711L_m$	$0,264L_m$
$0,723L_m$	$0,663L_m$
$0,276L_m$	$0,663L_m$



Shape optimization of heating chamber, shrinking device to knitted fabric with radiation condition witihin channel



Shape optimization of heating chamber, shrinking device to knitted fabric with radiation condition witihin channel

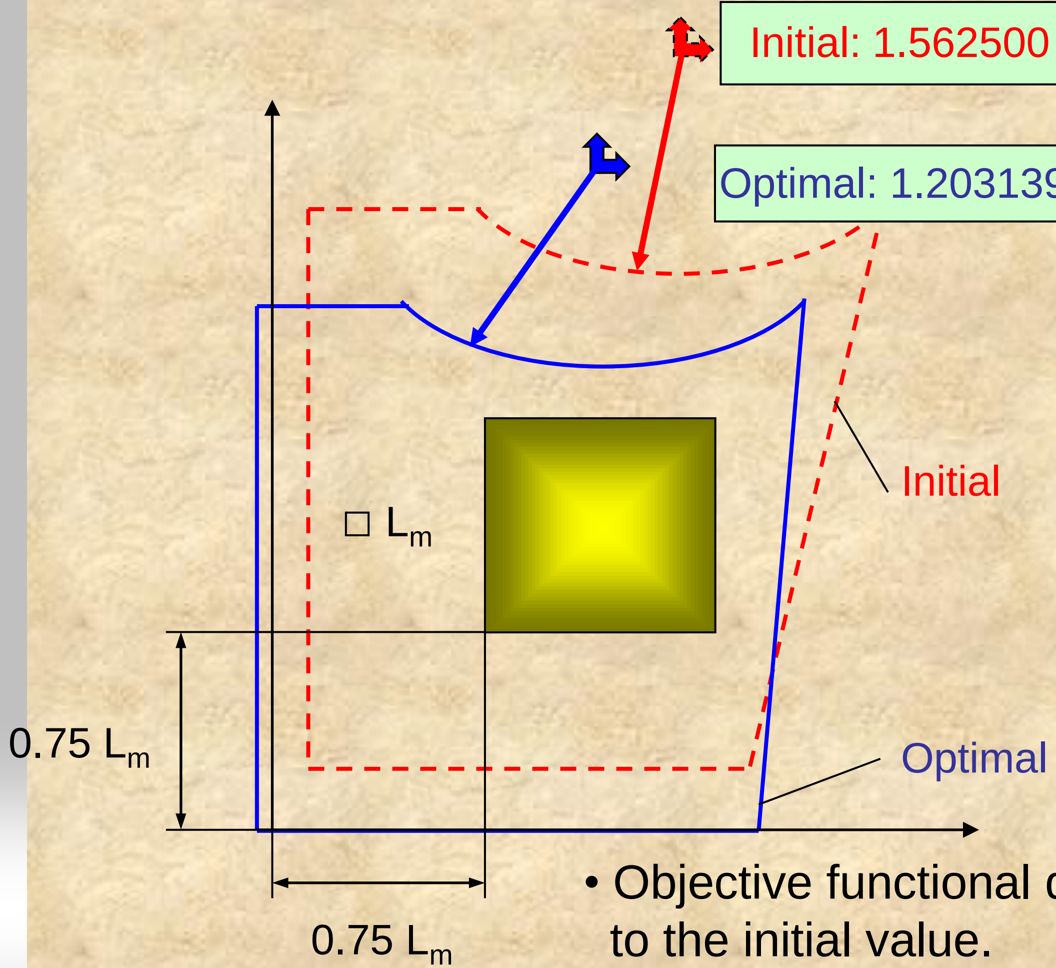


Shape optimization of heating chamber, shrinking device to knitted fabric with radiation condition witihin channel

- Upper boundary: shrinking process, constant heat flux density
- Other boundaries: convectional heat flux, T_∞=295K
- Hole: radiational heat flux, medium temperature T_∞=363K, wall emissivity ε=0,15; radiation absorption coefficient a=0,2;
- Radiational heat flux q_n^r is obtained by means of the weighted residual method.
- Design variables: coordinates of the vertex points on the external boundary + the centre of the input roller.
- Maximal angle of inclination of the side wall: 10°.
- Optimality conditions:

$$G = \left[\int_{\Gamma_q} \left(\frac{T}{T_0} \right)^n \right]^{\frac{1}{n}}, n = 25 \rightarrow \min \quad ; \quad C = C_0$$

Shape optimization of heating chamber, shrinking device to knitted fabric with radiation condition witihin channel



- Analysis stage: FE net of 4-nodal elements, direct approach to sensitivity analysis
- Synthesis stage: external penalty function.

- Objective functional decreases 10,57% in relation to the initial value.
- Hole closer to the boundary and in the middle (temperature distribution equalized on the part Γ_q)

Conclusions

- The available literature does not contain the following problems concerning the textile products / structures:
  - solutions of the similar shape optimization / identification problems by means of the material derivative concept and the sensitivity analysis.
  - introduction of the integral radiation condition in the form proposed according to Bialecki.
- The optimization in heat transfer problems is commercially important and frequent in textile structures.
- The variational approach of the FEM is applied, i.e. the objective functional has the clear physical interpretation and should describe the real problem.

Conclusions

- Shape optimization is the kind of adaptation to the environmental conditions = the analogy to the smart clothing.
- The discussed analysis can help to solve effectively the other heat transfer problems, cf.
  - modeling of the fireman suit during the firefighting action with the radiation and contact heat transfer,
  - transient diffusion problems.
- The analysis is economically important: the short calculation time, the little amount of work.

Conclusions

- Numerical implementation shows that the discussed methods can be the effective and fast tools to generate the structural shape (the identification) and model the textile structures (the optimization) subjected to the transient heat transfer.